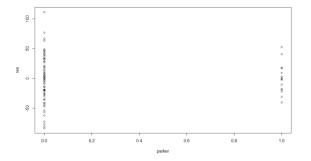
Collin DeVore Nicholas Kaukis STAT 3013 11/16/15

## Linear Regression Analysis

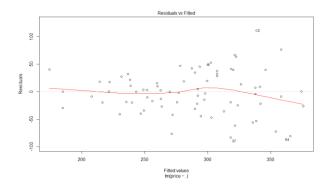
When analyzing the Homes model, I first used regression on the model so that I could get an idea of what it looked like with multicollinearity and have something to measure the progress of subsequent models off of. This gave an overall p-value of 8.906(e^-8), showing that there was some multicollinearity issues. I then created and analyzed the interactions that could be taking place between the factors using the "pairs" function on RStudio. Many of the plots did not seem to have much interaction between them, even though there were some that had an obvious interaction between them, such as Age and Agesq. These two factors seem to have a hook or parabolic shape between them. Also, Bathbed had an anticipated interaction with both Bath and Bed. None of the factors, however, had an obvious and unanticipated interaction with the other factors, so they were all left in at this point in the analysis. In order to find a model that more accurately represented the price, the p-values of the factors were calculated next to see what variables are fairly unrelated to price.

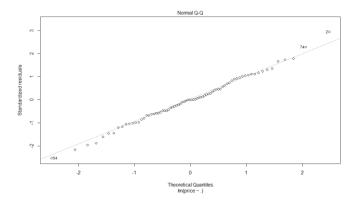
Looking at the p-values, there were some factors that had too high of p-values to be used. These variables were Age (0.24784), Garage (0.22460), Adams (0.30998), Crest (0.68266), and Parker (0.38942). Out of the five dummy variable sets regarding the different elementary schools that were nearby, only Harris (0.01685) and Edison (0.00187) had a significant impact on the price of the home, with Edison being more significant than Harris. This could mean that these two are prestigious schools, or that they are located near more expensive houses, though it does not imply that these schools are necessarily the cause of the price raise.

After looking at the p-values, I plotted the residuals in order to see if there is any problem regarding the data with the lower p-values. The residuals for the age and the garage appear to be spread well enough that there may just not be a correlation, but the residuals associated with the dummy variables seemed to be a bit heavy on one side and not well spread on the other, suggesting that they may not contain an accurate depiction of the dummy variables. An example of the problems with the residuals of the dummy variables can be seen with the Parker residuals graph shown below.



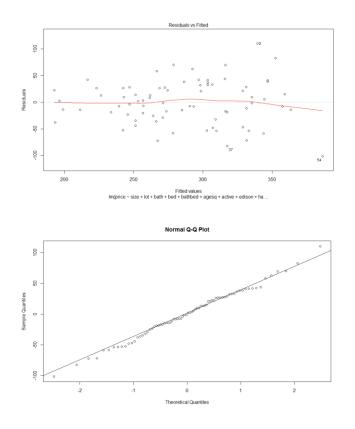
When I was done looking at all of the p- values separately, I looked at the graph of the residuals of the whole equation. The mean of the residuals had a fairly constant mean of zero, but the variance appears to have a funnel or cone shape that gets much larger to the right of the graph. The qq line kept showed that the residuals were normally distributed. The graphs of the residuals and the qq line are shown below.



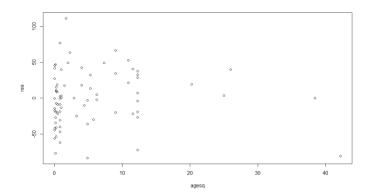


After looking at both the p-values and the residuals of the variables, a new model was used based on the factors with the lower p-values. The factors in this new model and their original p-values are Size (0.049), Lot (0.00518), Bath (0.01961), Bed (0.01961), Bathbed (0.0338), Agesq (0.01905), Active (0.01614), Edison (0.00187), and Harris (0.01685). These factors together give a p-value of 8.821(e^9). Although there is not much of a change in the overall p-value, it seems that the individual p-values are low enough to support the idea that these values fit the line the best. It may also signify that there are many values outside of this dataset that have not yet been explored. The new p-values for each of the factors are Size (0.004664), Lot (0.002776), Bath (0.061327), Bed (0.005313), Bathbed (0.026411), Agesq (0.091283), Active (0.005476), Edison (0.000315), and Harris (0.001001).

After the p-values of the new model were calculated, the new interaction plot was analyzed. The change in the factors gave more residuals that had a mean that appeared to equal zero. The variance was not well spread, though there appeared to be almost no interactions between factors excluding the Bathbed, Bed, and Bath factors' interaction. With many of the factors, there is an issue with constant variance, though this issue can be argued for each factor, and therefore these variables can be left in the formula. When all of the residuals are taken together, the variance appears to have improved due to the fact that the cone shape of the residuals are not as drastic. A model of the residuals when taken together is shown below, along with a model of the qq line, which showed that the residuals stayed normally distributed.



The rise in the p-value of Agesq suggests that a closer inspection of the factor is appropriate. Looking at the residuals of Agesq, five points can be seen that appear to be off to the side. These can skew the mean of the residuals and influence the values of the factor, which shows that it may not be a good depiction of the actual value of age squared. After that, there is the fact that there is a cluster of data points near the y axis, which does not appear to be significant since they are centered somewhere around 0. Even though these problems exist, the data is fairly distributed and there appears to be a constant variance, leading me to believe that the rest of the data points account and make up for this deviation of the later residuals. The graph of the residuals of Agesq is shown below.

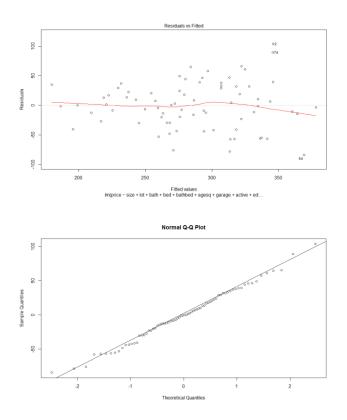


After analyzing the residuals associated with the Agesq factor, I analyzed the residuals associated with the other factors. For the dummy variables, there appears to be a larger spread and more variables on the right side, suggesting that they are better indicators for the model than the other dummy variables mentioned earlier. Some of the residuals for the other factors are questionable, such as the size, in which there is a slight chance that there may not be a constant variance. Most of the variance problems are caused by a single data point, however, so it still seems that this factor can be used as a fair predictor of price.

After the first two equations had been analyzed, I made one last equation using stepwise regression. Using the Aikaike Information Criterion, I obtained a new equation where the factors Size, Lot, Bath, Bed, Bathbed, Agesq, Garage, Active, Edison, and Harris were used. The interesting part of this equation is that all of the same factors are used in the second equation, except for garage, which must have had a higher p-value when only the other factors were used. This could be due to some multicollinearity issues that can be resolved when the other factors are taken out. Taken together, the Aikaike Information Criterion (AIC) of the first equation used is 800.4267, the AIC of the second equation used is 798.6595, and the AIC of the third equation is 795.6837. This shows a slight improvement from the original equation, though not by much.

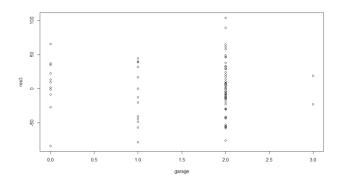
Looking at the residuals, the last equation fixes many of the issues associated with the first two models. The mean stays at about zero, the residuals follow the normality assumption as shown by the qq

plot, and there is almost a constant variance. There is still a slight cone shape, but, for the most part, that has been fixed, making it more usable. The residual plot and qq line are shown below.



After analyzing all of the residuals together, I analyzed them separately.

While there were still some issues associated with the spread of the residuals and some clustering problems, especially with Agesq which seemed to be having the same issues that it had earlier, the overall spread of these factors had improved. This may have occurred because stepwise regression took out the factors that were having the most issues, thus insuring that the factors that predicted the price best were used. In many of the residuals, there was not a constant variance, yet all other assumptions appeared to have been met. For instance, many had a mean that was equal to zero, though, to some extent, it could be argued that knowing one residual may help a person to better predict the location of another residual due to the shape of them. An example of one of the residual plots is shown below, in which the residuals of the factor Garage are shown.



The last procedure that was done was to make one last regression line model in which the factors were squared in order to see if there was any interaction between or within the factors during the analysis. Unfortunately, this yielded no results, showing that the optimal equation that can be used is equation three. None of the equations could account for most of the prices, showing that there may be other factors at work when deciding the price of the home. Many of these factors do still appear to have an effect, however. More studies can be done in order to see what other equation can influence the price of a home, though this is the best that can be done with the information provided.

## **Appendix**

## Code

attach(homesr)

pairs(~ ., data=homesr)

homes.model1 <- lm(price~., data = homesr)
summary(homes.model1)</pre>

plot (homesr\$age, homesr\$price)
plot (homesr\$agesq, homesr\$price)

res <- residuals(homes.model1) plot (size, res) plot (lot, res) plot (bath,res) plot (bath,res) plot (bathbed,res) plot (age,res) plot (agesq,res) plot (agesq,res) plot (active,res) plot (edison,res) plot (harris,res) plot (adams,res) plot (crest,res) plot (parker,res)

qqnorm(res) qqline(res)

homes.model2 <- Im(price~size+lot+bath+bed+bathbed+agesq+active+edison+harris, data=homesr) summary(homes.model2)

pairs(~size+lot+bath+bed+bathbed+agesq+active+edison+harris)

plot(homes.model1)
plot(homes.model2)

step (homes.model1)

homes.model3 <- step (homes.model1)
summary(homes.model3)</pre>

plot(homes.model3)

pairs(~size+lot+bath+bed+bathbed+agesq+garage+active+edison+harris)

AIC(homes.model1) AIC(homes.model2) AIC(homes.model3)

res2 <- residuals(homes.model2) plot (size, res2) plot (lot, res2) plot (bath,res2)

```
plot (bed,res2)
plot (bathbed, res2)
plot (age, res2)
plot (agesq,res2)
plot (garage, res2)
plot (active, res2)
plot (edison, res2)
plot (harris, res2)
plot (adams, res2)
plot (crest, res2)
plot (parker, res2)
res3 <- residuals(homes.model3)
plot (size, res3)
plot (lot, res3)
plot (bath, res3)
plot (bed,res3)
plot (bathbed, res3)
plot (age, res3)
plot (agesq,res3)
plot (garage, res3)
plot (active, res3)
plot (edison, res3)
plot (harris, res3)
plot (adams, res3)
plot (crest, res3)
plot (parker, res3)
qqnorm(res2)
qqline(res2)
qqnorm(res3)
qqline(res3)
homes.model4 <- Im(price~.^2,homesr)
summary(homes.model4)
Model Summaries
> homes.model1 <- lm(price~., data = homesr)</pre>
> summary(homes.model1)
Call:
lm(formula = price ~ ., data = homesr)
Residuals:
Min 1Q Median 3Q Max
-83.284 -22.628 -0.066 27.790 111.323
Coefficients:
                Estimate Std. Error t value Pr(>|t|)
337.6628 124.9621 2.702 0.00891 **
                                             2.702 2.009
(Intercept) 337.6628
                 58.7689
                                29.2569
                                                       0.04900 *
size
                                3.5731
47.9507
32.3252
13.6582
                                                       0.00518 **
lot
                 10.3619
                                             2.900
bath
                -98.7362
                                            -2.059
                                                       0.04376 *
                -77.4817
                                            -2.397
                                                       0.01961 *
bed
                                                       0.03380 *
bathbed
                 29.6573
                                              2.171
                                 3.2371
0.7571
                                                       0.24784 0.01905
                  3.7771
                                             1.167
age
                                              2.409
agesq
                  1.8236
                 10.6773
                                 8.7030
                                              1.227
                                                       0.22460
garage
                                                       0.01614 *
                 30.3572
                                12.2685
                                              2.474
active
                                                       0.00187 **
edison
                 59.2149
                                18.2076
                                              3.252
                 40.2345
                                16.3717
                                              2.458
                                                       0.01685 *
harris
                -28.8890
                                28.2176
adams
                                            -1.024
                                                       0.30998
```

crest -8.8819 21.6213 -0.411 0.68266 parker -13.933616.0736 -0.867 0.38942 Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1 Residual standard error: 42.37 on 61 degrees of freedom Multiple R-squared: 0.5989, Adjusted R-squared: 0.5068 F-statistic: 6.505 on 14 and 61 DF, p-value: 8.906e-08 > homes.model2 <- lm(price~size+lot+bath+bed+bathbed+agesq+active+edison+harris, data=</pre> homesr) > summary(homes.model2) Call: lm(formula = price ~ size + lot + bath + bed + bathbed + agesg + active + edison + harris, data = homesr) Residuals: Median Min 10 Мах -23.721 -101.578 27.577 110.508 0.133 Coefficients: Estimate Std. Error t value Pr(>|t|) 108.943 27.978 2.929 0.004664 \*\* 2.550 0.013086 \* 319.093 (Intercept) 71.358 size 3.108 0.002776 \*\* lot 10.617 3.416 -82.136 -82.000 hath 43.148 -1.904 0.061327 -2.883 0.005313 \*\* 28.443 bed bathbed 27.523 12.119 2.271 0.026411 \* 1.237 1.714 0.091283 agesq 0.722 2.872 0.005476 \*\* 31.853 11.090 active 3.803 0.000315 \*\*\* 62.787 16.509 edison 14.978 3.444 0.001001 \*\* harris 51.584 signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1 Residual standard error: 43 on 66 degrees of freedom Multiple R-squared: 0.553, Adjusted R-squared: 0.492 F-statistic: 9.072 on 9 and 66 DF, p-value: 8.821e-09 summary(homes.mode13) Call: lm(formula = price ~ size + lot + bath + bed + bathbed + agesq + garage + active + edison + harris, data = homesr) Residuals: 1Q Median 3Q Min Мах -84.234 -24.091 -0.581 29.075 104.106 Coefficients: Estimate Std. Error t value Pr(>|t|) (Intercept) 322.7285 106.2569 3.037 0.003435 \*\* 27.8541 2.140 0.036112 \* 59.6082 size 2.745 0.007815 \*\* lot 9.3038 3.3893 -92.8312 42.3867 -2.190 0.032111 \* bath -2.869 0.005549 \*\* -79.6453 27.7610 bed bathbed 29.5347 11.8574 2.491 0.015308 \* 1.4557 0.7118 2.045 0.044878 \* agesq 2.097 0.039875 \* garage 16.2220 7.7353 10.9913 27.7465 2.524 0.014040 \* active 61.9646 3.848 0.000275 \*\*\* 16.1050 edison 49.9654 14.6272 3.416 0.001100 \*\* harris Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1 Residual standard error: 41.93 on 65 degrees of freedom Multiple R-squared: 0.5813, Adjusted R-squared F-statistic: 9.025 on 10 and 65 DF, p-value: 4e-09 Adjusted R-squared: 0.5169

<pre>&gt; homes.model4 &lt;-lm(price~.^2,homesr) &gt; summary(homes.model4)</pre>							
Call: lm(formula =	price ~ .^2,	data = hor	mesr)				
Residuals:							
-9.820e-15 7	-	3 766e-15 5		5 -3.992e-16			
-2.241e-15 -3 13	8 .763e-14 8. 14	227e-15 7 15	10 178e-15. 16	11 -3.850e-14 17			
5.925e-14 -1 19	—			-1.884e-15	18 1.347e-13 24		
-4.215e-13 -6		457e-16 -1 27					
	.569e-14 -2.						
1.733e-16 -1 37				-2.574e-15 41			
6.612e-15 -6 43					1.460e-14 48		
-4.235e-14 -3 49		972e-14 -1 51			-9.345e-14 54		
	.469e-16 -1.				1.085e-15 60		
-3.346e-15 -1 61	.597e-14 3.	316e-14 1 63	.044e-14 64	5.625e+00 65	-4.333e+00 66		
-5.625e+00 -3 67	.333e-01 4.	667e+00 7 69	.965e-15 70	-1.786e-14 71			
3.690e-13 -5 73	74	75	76	3.233e-16	7.426e-16		
4.438e-15 2							
Coefficients:		Fined becaus Std. Error					
(Intercept)	30674.55	69920.30	0.439	0.69055			
size lot	-42013.34 -3865.14	29343.43 4652.54	-1.432 -0.831	0.24761 0.46705			
bath	16682.49	36127.34	0.462	0.67567			
bed bathbed	-9210.35 -507.76	20147.24 9807.71	-0.457 -0.052	0.67863 0.96197			
age	-9862.19	4350.87	-2.267	0.10825			
agesq	1955.93	830.69	2.355	0.09989 .			
garage active	16761.83 -6524.18	6086.66 1680.91					
edison	20869.93	7045.49		0.05944 .			
harris	154178.77	51793.94	2.977	0.05875 .			
adams crest	11430.87 3630.37	4831.29 6095.82	2.300	0.09886 . 0.59341			
parker	-1985.29	5832.97	-0.340	0.75603			
size:lot size:bath	1226.20 6197.07	293.55 14029.61	4.177 0.442	0.02499 * 0.68860			
size:bed	10011.33	8515.16	1.176	0.32452			
size:bathbed	-2068.51	4061.57	-0.509	0.64565			
size:age size:agesg	-594.70 184.12	146.81 27.62	-4.051 6.666	0.02709 * 0.00688 **			
size:garage	2840.82	692.95	4.100	0.02626 *			
size:active size:edison	3127.10	785.70	3.980	0.02838 * 0.01682 *			
size:harris	-1987.39 -6804.34	410.57 1895.59	-4.841 -3.590	0.03703 *			
size:adams	-5863.16	2515.39	-2.331	0.10206			
size:crest size:parker	1426.01 -5981.39	2165.55 1683.58	0.658 -3.553	0.55724 0.03802 *			
lot:bath	-384.98	2310.86	-0.167	0.87828			
lot:bed	378.31	1433.43	0.264	0.80893			
lot:bathbed lot:age	18.01 36.92	707.46 33.22	$0.025 \\ 1.111$	0.98129 0.34746			
lot:agesq	17.87	24.12	0.741	0.51243			

lot:garage lot:active lot:edison	374.14 -157.72 910.82	112.38 57.10 265.18	3.329 -2.762 3.435	0.04474 * 0.07002 . 0.04139 *
lot:harris lot:adams lot:crest lot:parker	508.00 281.50 2013.32 -143.24	$137.70 \\ 126.03 \\ 686.57 \\ 131.72 \\ \cdots$	3.689 2.234 2.932 -1.087	0.03454 * 0.11162 0.06088 . 0.35640
bath:bed bath:bathbed bath:age bath:agesq bath:garage	NA -457.60 5107.28 -1654.59 -11795.23	NA 201.29 2127.53 570.97 3622.55	NA -2.273 2.401 -2.898 -3.256	NA 0.10759 0.09583 . 0.06261 . 0.04727 *
bath:active bath:edison bath:harris bath:adams	-104.81 -5932.15 -70086.72	612.53 2849.01 23770.60 NA	-0.171 -2.082 -2.948 NA	0.87502 0.12873 0.06010 .
bath:crest bath:parker bed:bathbed bed:age	-6642.94 7948.23 -97.59 2299.10	1467.22 4124.82 60.03 1091.01	-4.528 1.927 -1.626 2.107	0.02016 * 0.14962 0.20248 0.12569
bed:agesq bed:garage bed:active bed:edison	-680.43 -5979.90 273.35 -6674.89	257.98 1878.84 253.91 2498.34	-2.638 -3.183 1.077 -2.672	0.07782 . 0.04999 * 0.36055 0.07558 .
bed:harris bed:adams bed:crest bed:parker bathbed:age	-46978.71 NA -887.93 4753.38 -1263.95	16484.12 NA 594.57 2466.37 562.50	-2.850 NA -1.493 1.927 -2.247	0.06511 . NA 0.23217 0.14956 0.11024
bathbed:agesq bathbed:garage bathbed:active bathbed:edison	465.56 3078.07 48.62 1788.62	166.24 938.80 143.68 939.69	2.247 2.800 3.279 0.338 1.903	0.06783 . 0.04647 * 0.75739 0.15312
bathbed:harris bathbed:adams bathbed:crest bathbed:parker	23010.28 NA 187.81 -2651.38	7943.94 NA 370.05 1312.46	2.897 NA 0.508 -2.020	0.06268 . NA 0.64674 0.13664
age:agesq age:garage age:active age:edison	73.89 430.62 -261.29 -332.89	$\begin{array}{r} 22.30 \\ 114.42 \\ 68.35 \\ 95.12 \\ 46.99 \end{array}$	3.313 3.764 -3.823 -3.500	0.04530 * 0.03281 * 0.03151 * 0.03949 * 0.45319
age:harris age:adams age:crest age:parker agesg:garage	40.40 NA NA 51.72 89.36	40.99 NA NA 54.63 35.23	0.860 NA 0.947 2.537	0.43319 NA NA 0.41360 0.08493.
agesq:active agesq:edison agesq:harris agesq:adams	NA NA NA NA	NA NA NA NA	NA NA NA NA	NA NA NA NA NA
agesq:crest agesq:parker garage:active garage:edison	NA NA NA NA	NA NA NA	NA NA NA	NA NA NA NA
garage:harris garage:adams garage:crest garage:parker active:edison	NA NA NA NA	NA NA NA NA	NA NA NA NA	NA NA NA NA
active:harris active:adams active:crest active:parker	NA NA NA NA	NA NA NA NA	NA NA NA NA	NA NA NA NA NA
edison:harris edison:adams edison:crest edison:parker	NA NA NA NA	NA NA NA NA	NA NA NA NA	NA NA NA NA
harris:adams harris:crest harris:parker adams:crest adams:parker	NA NA NA NA	NA NA NA NA	NA NA NA NA	NA NA NA NA

crest:parker	NA	NA	NA	NA	
Signif. codes:	0 '***' 0.001	'**' 0.01	'*' 0.0	5 '.' 0.1	''1

Residual standard error: 5.886 on 3 degrees of freedom Multiple R-squared: 0.9996, Adjusted R-squared: 0.9905 F-statistic: 109.4 on 72 and 3 DF, p-value: 0.00121

## **Stepwise Regression**

```
> homes.model3 <- step (homes.model1)</pre>
Start: AIC=582.75
price \sim size + lot + bath + bed + bathbed + age + agesq + garage +
     active + edison + harris + adams + crest + parker
             Df Sum of Sq R55
1 302.9 109809 580.96
- crest
- parker
                     1349.0 110855 581.68
                     1881.6 111387 582.04
2444.0 111950 582.43
2702.0 112208 582.60
              1
- adams
              1
- age
- garage
              1
                     109506 582.75
7243.5 116749 585.62
7611.5 117117 585.86
<none>
- size
              1
- bath
              1
- bathbed
              1
                     8464.2 117970 586.41
                    10313.9 119820 587.59
10415.1 119921 587.65
- bed
              1
  agesq
              1
_
- harris
                    10842.2 120348 587.92
              1
                   10991.2 120497 588.02
15097.1 124603 590.56
18987.2 128493 592.90
- active
              1
_
  lot
              1
- edison
              1
Step: AIC=580.96
price ~ size + lot + bath + bed + bathbed + age + agesq + garage +
     active + edison + harris + adams + parker
             Df Sum of Sq
                                  RSS
                                           AIC
                     1088.5 110897 579.71
- parker
              1
- adams
              1
                     1679.8 111489 580.11
                     2387.2 112196 580.59
109809 580.96
- age
              1
<none>
                     3273.2 113082 581.19
6948.1 116757 583.62
7406.3 117215 583.92
- garage
              1
- size
              1
- bath
              1
- bathbed
              1
                     8289.3 118098 584.49
                    10151.8 119961 585.68
- bed
              1
                    10237.6 120046 585.73
11394.5 121203 586.46
  agesg
              1
              1
  active
- harris
              1
                    12196.6 122005 586.96
                    14794.5 124603 588.56
20834.3 130643 592.16
- lot
              1
- edison
              1
Step: AIC=579.71
price \sim size + lot + bath + bed + bathbed + age + agesg + garage +
     active + edison + harris + adams
             Df Sum of Sq RSS ALC
1 1498.2 112395 578.73
- adams
- age
              1
                     2011.5 112909 579.07
                     110897 579.71
3666.5 114564 580.18
6304.2 117202 581.91
<none>
- garage
              1
- sīze
              1
                     9400.7 120298 583.89
              1
- agesq
                    10589.0 121486 584.64
10689.6 121587 584.70
_
              1
  active
- bath
              1
                    12934.4 123832 586.09
- bathbed
              1
- lot
              1
                    13850.1 124747 586.65
```

15696.2 126593 587.77 16508.4 127406 588.25 24361.9 135259 592.80 - bed 1 1 - harris - edison 1 Step: AIC=578.73 Df Sum of Sq RSS AIC 1 1905.4 114301 578.01 - age <none> 112395 578.73 4403.5 116799 579.65 7219.8 119615 581.46 8799.0 121194 582.46 1 - garage - size 1 - agesq 1 9294.7 121690 582.77 10938.1 123334 583.79 11495.9 123891 584.13 - bāth 1 - active 1 - bathbed  $\overline{1}$ 14198.6 126594 585.77 14612.8 127008 586.02 17810.5 130206 587.91 27910.3 140306 593.58 - bed 1 -1 lot - harris 1 - edison 1 Step: AIC=578.01 price ~ size + lot + bath + bed + bathbed + agesq + garage + active + edison + harris Df Sum of Sq RSS AIC 114301 578.01 7356.1 121657 580.75 7733.8 122035 580.98 <none> - agesq 1 -1 garage 7733.8 122035 580.98 8053.2 122354 581.18 8434.6 122735 581.42 10910.0 125211 582.93 11206.1 125507 583.11 13250.4 127551 584.34 14473.9 128775 585.07 20518.9 134820 588.55 26031.6 140332 591.60 1 size - bath 1 bathbed 1 -1 active - 1ot

1

1

1

1

- bed

- harris

- edison